The Geostrophic Assumption

The Horizontal Equation of Motion and the Navier-Stokes Equation can be simplified for large scale flow in the atmosphere and the ocean with the assumption that friction and viscosity effects are small on an order of magnitude basis.

In that case, the horizontal frictionless equation of motion in concept form reduces to:

Total Acceleration of (Air/Water) Parcel = **Pressure Gradient Acceleration** and **Coriolis Acceleration**

A. _Geostrophic Wind Equation

The pressure gradient terms can be transformed by use of the hydrostatic equation into height gradient terms. This allows the magnitude of the speed to be related to height gradients. We'll do that transformation in ERTH 465. For now, we simply are inserting those expressions into the Horizontal Equation of Motion in natural coordinates.

$$\frac{\Delta V}{\Delta t} = \left(-g\frac{\Delta z}{\Delta s} - g\frac{\Delta z}{\Delta n}\right) - 2\Omega V sin\phi \qquad (1)$$

Let's apply this equation to the case we examined in the last class, as shown in Figure 1 below.

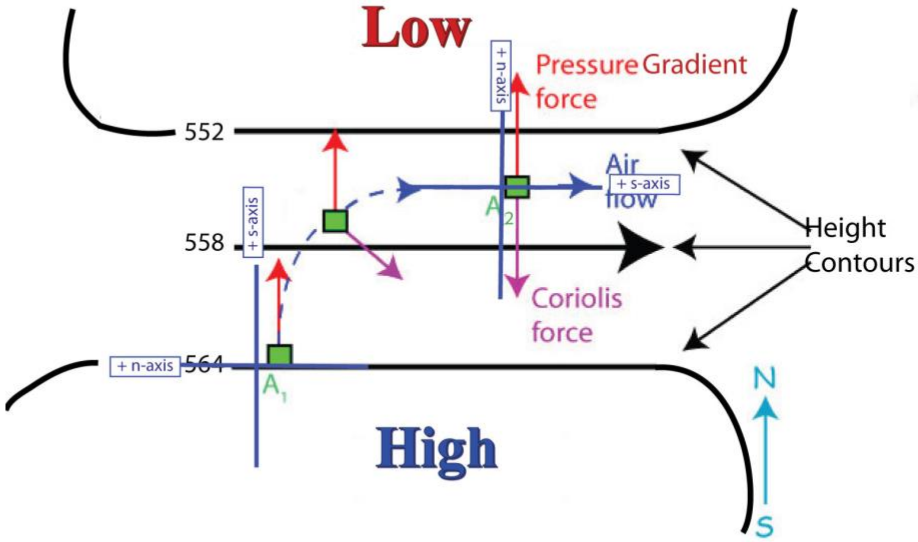


Figure 1*: Balance of forces or accelerations in the horizontal Equation of Motion for the case of an initially stationary air parcel at the 500 mb level in the middle latitudes. A_1 indicates the initial state and A_2 the final state, as discussed in class.*

At the outset, the parcel shown at A_1 is at rest with respect to the surface of the earth and the pressure gradient acceleration term acts northward. The natural coordinate system is set up so that the positive s axis is tangent to the acceleration vector with the positive n axis at a counterclockwise right angle to it. On the right side of Equation 1 there can be no Coriolis Acceleration since there is no motion, and there is no height gradient along the n axis. Hence, Equation (1) simplifies to

$$\frac{\Delta V}{\Delta t} = -g\frac{\Delta z}{\Delta s} \qquad (2)$$

which is a mathematical equivalent of the first rule of thumb with respect to motion we learned in this class: in the absence of other effects, air accelerates from regions of higher values of pressure (in this case, heights) to lower values, at right angles to the contours.

The final state we discussed in class is shown at A_2. In this case we saw that the wind is blowing parallel to height contours. Hence, there is no variation of height along the wind streamlines, in that case, and $\Delta z/\Delta s$ is zero.

In addition, if air is moving parallel to the isobars or height contours and there is a perfect balance between the pressure gradient acceleration and the Coriolis acceleration, the total acceleration experienced by the air parcel is zero, although the wind vector will be large. This will remain true as long as the field of isobars or height contours is not changing (the pressure gradients do not change).

In that restrictive circumstance, the net acceleration on the left hand side of the expression is zero.

$$0 = -g\frac{\Delta z}{\Delta n} - 2\Omega V sin\phi \qquad (3)$$

Solving (3) for the wind speed, V, termed the "geostrophic wind" velocity, we get a relatively simple equation. “Geostrophic” means in “earth balance” referring to the balance between pressure gradient acceleration and Coriolis acceleration ultimately achieved by wind and water parcels on the earth (but, in reality, in any atmosphere and on any planet).

It provides insight into the flow that develops that is due to the balance of the pressure gradient acceleration normal to the flow and Coriolis acceleration. Because we made some simplifications that are realistic only in certain circumstances, called the geostrophic approximation, the wind symbol obtained by solving for V in (3) normally has a subscript "g" to indicate that these simplifications were made.

$$V_g = -\frac{g}{f}\frac{\Delta z}{\Delta n} \qquad (4)$$

where f=2$\Omega sin\phi$. f is referred to in a number of ways. In this case, it is called the Coriolis parameter. Equation (4) is known as the **geostrophic wind equation**. It is elegant in its simplicity and, because of that, students are very fond of it. It says that if the wind is geostrophic, at a given latitude the wind speed is only related to the height (pressure) gradient.

B. A Measure of How Realistic the Geostrophic Assumption Is: The Rossby Number

It doesn't take much consideration to realize that the Geostrophic Assumption is unrealistic in many circumstances, many of them obvious, and some subtle. In the real atmosphere (and ocean) there are almost always small accelerations.

Notice in Figure 1 that the Coriolis Acceleration vector increased as the wind speed increased. At the same time the total acceleration decreased to, finally, zero in the last state. In other words, the ratio of the total acceleration to Coriolis acceleration gets smaller and smaller as the situation approaches geostrophic balance.

This ratio is called the Rossby Number, and can be used to decide to what extent the "geostrophic wind" corresponds to the real wind, as a function of scale. Ratios of 0.1 or less indicates that the total acceleration is one to two orders of magnitude smaller than the Coriolis acceleration and, thus, the total acceleration can be dropped on an order of magnitude basis: the wind flow will be tangent to the isobars or height contours (see Table 1).

Values approaching 1 or more indicate that the total acceleration experienced by an air parcel is very large, and cannot be dropped out of the equation. In those cases, using the idea of the geostrophic wind to explain what you see on a weather map will produce confusion. In those cases, it will appear that air is moving at right angles to the isobars from higher values of pressure to lower values of pressure. For example, the Rossby Number is very very large the closer one gets to the Equator where Coriolis Acceleration is zero.

In Middle Latitudes (Middle Troposphere) **Scale of Circulation (Distance Scale; no Friction)**	**Ratio of Total Acceleration to Coriolis Acceleration (Rossby Number)**	**Real Wind Generally Explained by Geostrophic Wind?**
10,000 km	0.01	Yes
1,000 km	0.1	Yes
100 km	1.0	Not really
10 km	3.0	No
1 km	4.0	No

Table 1: *Characteristic Rossby Number for different scales of circulation and evaluated in the middle Troposphere. Red shading indicates that the geostrophic assumption is invalid and air motion will be at right angles to isobars. Green shading indicates scales at which the geostrophic wind assumption is valid and the geostrophic wind can be considered a valid approximation of the real wind.*

C. The "Real" Wind

Another way to look at this is that real wind is always made up of a portion that is geostrophic and a portion that is not geostrophic (called AGEOSTROPHIC). Here's a simple algebraic expression that summarizes this concept. In this case, the symbol, V, indicates the real (two dimensional) horizontal wind in natural coordinates.

$$V = V_{geo} + V_{ageo} \qquad (3)$$

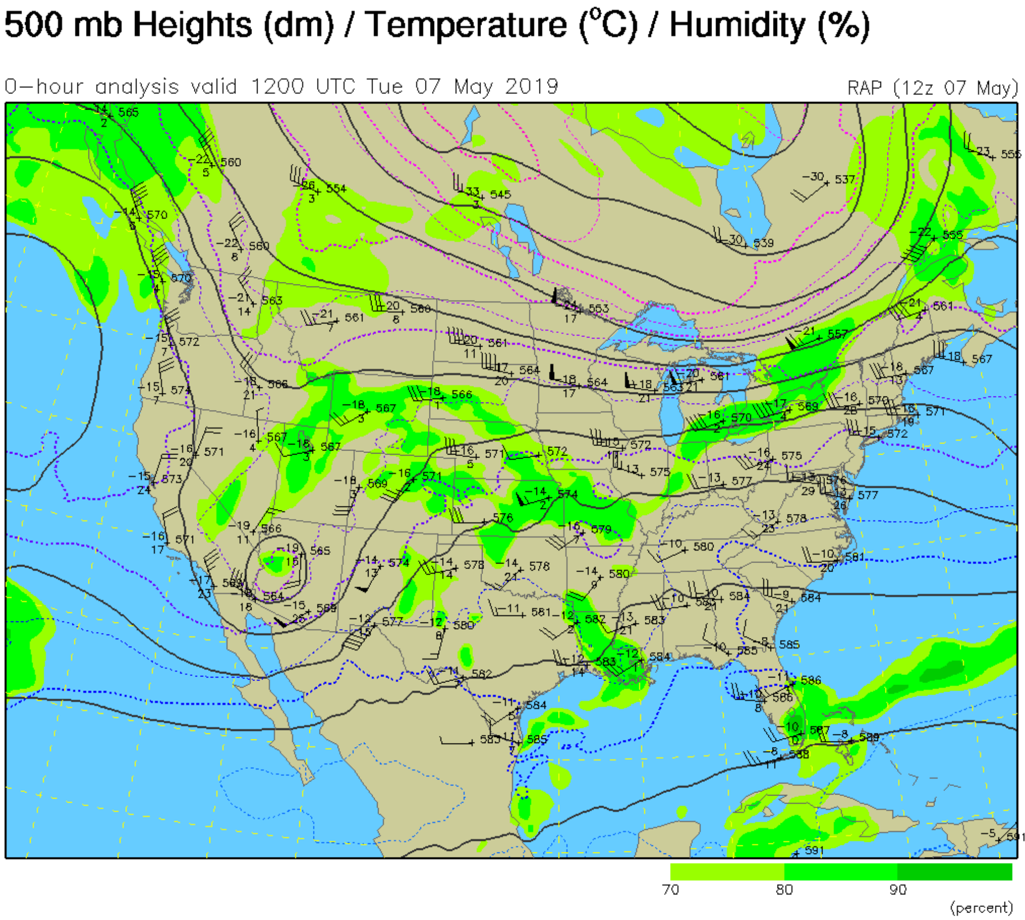
It turns out that even at the synoptic and macroscales, there are situations in which the total acceleration (ageostrophic wind) is significant. These situations generally occur in certain portions, or levels, of the troposphere. Also, there is a portion of the troposphere, or level, in which the total acceleration (ageostrophic wind) is small. There the wind will appear to be flowing parallel to the isobars (or height contours) with a magnitude exactly determined by the pressure gradient. It also turns out that the ageostrophic accelerations are related to the horizontal divergence patterns in the atmosphere (see Table 2).

Level of the Troposphere	$\mathbf{V_{geo}}$	$\mathbf{V_{ageo}}$	**Real Wind "Looks" Like Geostrophic Wind?**	**Net Acceleration**	**Hori-zontal Divergence**
Upper (300-200 mb level)	**Yes**	**Large**	**Somewhat**	**Large**	**"Large"**
Middle (600 to 450 mb)	**Yes**	**None to small**	**Yes**	**None to small**	**None to small**
Lower (Sfc to 925 mb)	**Yes**	**Large**	**Somewhat**	**Large**	**"Large "**

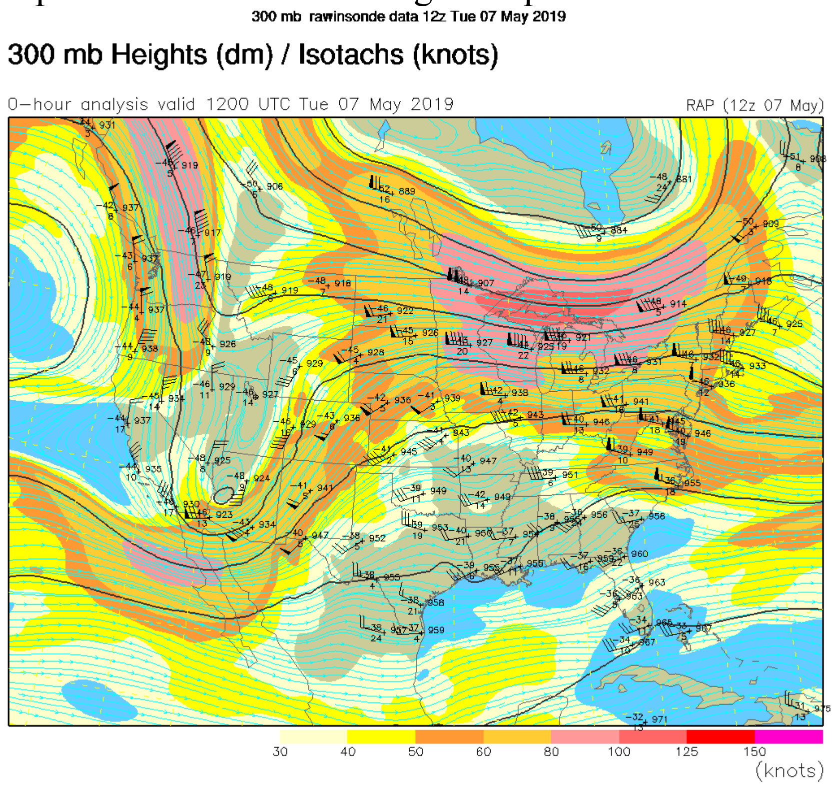
Table 2: *Qualitative evaluation of the presence of the geostrophic wind and the ageostrophic wind in the lower, middle, and upper troposphere. Only in the middle troposphere (green) is the geostrophic wind similar to the actual wind observed. At the other levels ageostrophic accelerations and winds are large contributions so that the actual wind cannot be approximated by the geostrophic wind realistically.*

Thus, students who embrace the concept of the geostrophic approximation "love" the 500 mb chart, because there the wind appears to flow parallel to the contours and in direct proportion to the pressure (or height) gradient. That's the level at which the geostrophic approximation corresponds most closely to the real wind.

500 mb rawinsonde data 12z Tue 07 May 2019



This is not quite the case on [200 and 300 mb charts](). It's true that even the largest ageostrophic motions there are relatively small compared to the geostrophic motion. So, at first glance it will appear that the wind is in geostrophic balance. A closer look will reveal areas of cross contour flow, and, also, areas in which the wind speeds do not match what one would expect from solution of the geostrophic wind relation.



Finally, at the ground, there is also substantial cross contour flow. This "disruption" of the geostrophic wind balance occurs chiefly because of friction. That's why the surface wind will most closely resemble the geostrophic wind in regions in which frictional effects are minimal (e.g., over oceans, over flat featureless plains).