Divergence and Resulting Mid-tropospheric Vertical Velocity and Surface Pressure Tendency for Waves in the Upper Troposphere

We have just finished discussing how to visualize horizontal divergence and to conceptualize where divergence is found with respect to the wave-like disturbances in, say, the upper troposphere. This horizontal divergence is then connected, via the Equation of Continuity, to the creation of dynamic lows at the surface. One good example that we've discussed is the surface, baroclinic wave cyclone, illustrated by the cartoon in Figure 1.

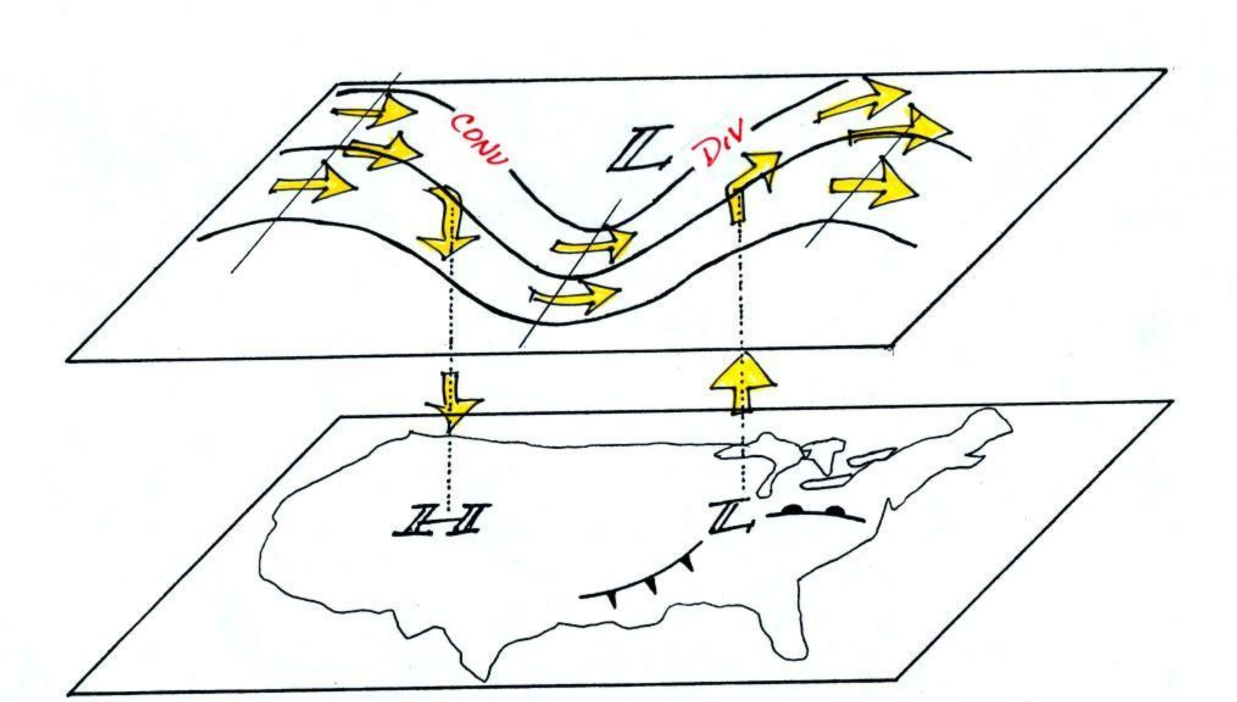

Figure 1: Illustration of the relationship of surface wave cyclones to the divergence aloft east of a short wave trough in the upper troposphere.

The horizontal divergence shown in Fig. 1 in the upper troposphere is related to the algebraic sum of the speed divergence and diffluence occurring east of the trough axis, which in turn relates to the geometry of the height pattern at that level.

The height pattern east of the trough shown in Fig. 1 is typical, and the geometry can be worked out. This derivation is something we might do in ERTH 465 or ERTH 565. For now, I just state the relationship between horizontal divergence east of the trough axis to the ridge axis at a given latitude for a given average wind speed at that level:

$$DIV_h \approx k \ \frac{4VA}{L} \quad (1)$$

where k is a constant, V is the mean wind speed in the region, A is the amplitude of the disturbance, and L is the wavelength.

This Equation (1) states that the horizontal divergence is directly responsible to the wind speed, the amplitude of the disturbance, and inversely related to wavelength.

In other words, short wave disturbances in the middle and upper troposphere have much more significant divergence (and convergence) associated with them than long wave disturbances. For example, Rossby waves are basically non-divergent.

Equation (1) explains why much more significant dynamic lows (intense wave cyclones, for example), occur in the winter than in the summer, because the mean zonal wind speed in the upper troposphere is much stronger during the winter. The expression also explains why, in the process of baroclinic instability as the amplitude of the disturbance in the middle and upper troposphere gets larger, more divergence occurs at the inflection point between the trough and the ridge.

Since equation (1) relates to vertical motion at the Level of Non-divergence via Dine's Compensation, the equation can be rewritten into one that has lofting (or subsidence) on the left side of the equals sign. The net result of all of this with respect to wavelength for a given wind speed and amplitude is summarized in Tables 1 and 2 below.

	Magnitude of the Horizontal Divergence at Inflection Point	Linked Pressure Change at the Surface	Upward Motion at LND
Short Waves	Large	Large	Larger
Long Waves	Small	Small	Smaller

Table 1: Implication of Equation (1) for a given zonal wind speed in upper troposphere and its relation to vertical motion in mid-Troposphere and pressure change at the ground

	Magnitude of the Horizontal Divergence at Inflection Point	Linked Pressure Change at the Surface	Upward Motion at LND
Higher Amplitude	Larger	Larger	Larger
Higher Wind Speed	Larger	Larger	Larger

Table 2: Implication of Equation (1) for a given wavelength and its relation to vertical motion in mid-Troposphere and pressure change at the ground