Department of Earth & Climate Sciences
San Francisco State University

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Name ______________________

Homework #6 Key: First Law of Thermodynamics
and the Dry Adiabatic Lapse Rate (100 pts)
(Due Wednesday 14 March)

Answer in complete sentences on separate sheets. Please don't jam your answers in on this sheet.

The First Law of Thermodynamics in mathematical form is

$$\frac{\Delta T}{\Delta t} = \frac{1}{c_p}\frac{\Delta q}{\Delta t} + \frac{1}{c_p \rho}\frac{\Delta p}{\Delta t} \qquad (1)$$

where Δq is sensible heat, c_p is the specific heat constant.

The equation above is the prognostic version of the First Law of Thermodynamics. This means it can be used to forecast temperature changes experienced by air parcels.

The temperature change simply due to the adiabatic term is

$$\frac{\Delta T}{\Delta t} = \frac{1}{c_p \rho}\frac{\Delta p}{\Delta t} \qquad (2)$$

The hydrostatic equation is

$$\frac{\Delta p}{\Delta z} = -\rho g$$

(3a, b)

$$\Delta p = -\rho g \Delta z$$

1. Substitute Equation (3b) into equation (2).

$$\frac{DT}{Dt} = -\frac{g}{c_p}\frac{Dz}{Dt}$$

2. Simplify your result in the last step by substitution of the definition of the dry adiabatic lapse rate.

$$\Gamma_a = -\frac{g}{c_p}$$

$$\frac{\Delta T}{\Delta t} = \Gamma_a \frac{\Delta z}{\Delta t}$$

2. Simplify your result in the last step by substitution of the mathematical symbol for the vertical wind component, w.

$$w = \frac{\Delta z}{\Delta t}$$

$$\frac{\Delta T}{\Delta t} = \mathrm{w}\Gamma_a$$

3. What is the cooling rate (in units of degrees C/minute) for air moving upward with a speed of 10 meters/second? (Show all steps and calculations).

   You can assume that the dry adiabatic lapse rate is -1^{o}C/100 m.

$$\frac{\Delta T}{\Delta t} = \left(\frac{10m}{s}\right)\left(\frac{-1^{0}}{100m}\right)\left(\frac{60s}{1min}\right)$$

$$\frac{\Delta T}{\Delta t} = \left(\frac{-6^{0}}{min}\right)$$

This means that air lofted will cool considerably for each minute it moves upward. In this case, it will have moved upward around 600 m (~1800 feet). The concept of the adiabatic term is that air cools when it expands and warms when it contracts. Since air is moving upwards into regions of lower pressure, the air parcel's density will decrease, and the temperature should go down. This is consistent with the results obtained here.