Department of Earth & Climate Sciences Spring 2018
San Francisco State University Name ______________________

Homework #7 Key: Isentropes and Isohumes (100 pts)
(Due Wednesday 28 March)

For numerical problems show all steps and work on separate sheets. For conceptual problems, answer in complete sentences on separate sheets. Please don't jam your answers in on this sheet.

Poisson's Relation is given as Equation (1). The Clausius-Clapeyron equation is given as Equation (2). The relationship between saturation mixing ratio, saturation vapor pressure, and pressure is given in Equation (3).

$$\theta = T\left(\frac{1000\ mb}{p}\right)^{0.286} \qquad (1)$$

$$e_s = (0.6113\ kPa)^{\left[6139K\left(\frac{1}{273.15K}-\frac{1}{T}\right)\right]} \qquad (2)$$

$$w_s = \frac{(622\ g\ kg^{-1})e_s}{P-e_s} \qquad (3)$$

1. (35 pts) In the handout covering Poisson's Relation, using equation (1) we computed the 500 mb temperature that corresponded to a 1000 mb temperature of 20ºC (293ºK).
   a. Plot these two points as black X's on the attached Stüve Diagram. Create an isentrope by conn these two points with a straight line and label with the correct potential temperature in K. (15 pts)
   b. Using the same procedure, compute the 500 mb temperature that corresponds to a 1000 mb temperature of 30ºC (303ºK). Plot these two points as black X's on the attached Stüve Diagram. Create an isentrope by connecting these two points

with a straight line and label with the correct potential temperature in K. (20 pts)

$$303 = T(2)^{0.286}$$

303K=1.2195 T

T=248.5 K=-24.5 C

2. (35 pts) In the handout covering the Clausius-Clapeyron Equation, we used equation (2) to calculate the saturation vapor that corresponded to a 1000 mb temperature of 30^{o}C (303^{o}K). We then used equation (3) to calculate the saturation mixing ratio that corresponded to a 1000 mb temperature of 30^{o}C (303^{o}K).

   That value was 28.4 g kg^{-1}. Plot this value as a red X on the attached Stüve Diagram at the point of 30^{o}C at 1000 mb. (5 pts)

   a. Using Equation (2), compute the saturation vapor pressure for a temperature of 18.5 ^{o}C. (10 pts)

$$\boldsymbol{e_s = (0.6113\ kPa)^{\left[-6139K\left(\frac{1}{273.15K}-\frac{1}{T}\right)\right]}}$$

$$\boldsymbol{e_s = (0.6113\ kPa)^{[-1.588]} = 2.185\ kPa}\text{=21.85 mb}$$

   b. Using Equation (3), use the value of saturation vapor pressure you obtained in (a) to compute the saturation mixing ratio at 500 mb. Plot this value as a red X on the attached Stüve Diagram at the spot corresponding to 18.5^{o}C at 500 mb. (15 pts)

$$\boldsymbol{w_s = \frac{(622\ g\ kg^{-1})e_s}{P - e_s}}$$

$$w_s = \frac{(622\ g\ kg^{-1})21.85\ mb}{500\ mb\ - 21.85\ mb} = \frac{13590.7\ g\ kg^{-1}}{478.15} = 28.4\ g\ kg^{-1}$$

c. Draw the 28 g/kg isohume as a dashed based upon your results. (5 pts)

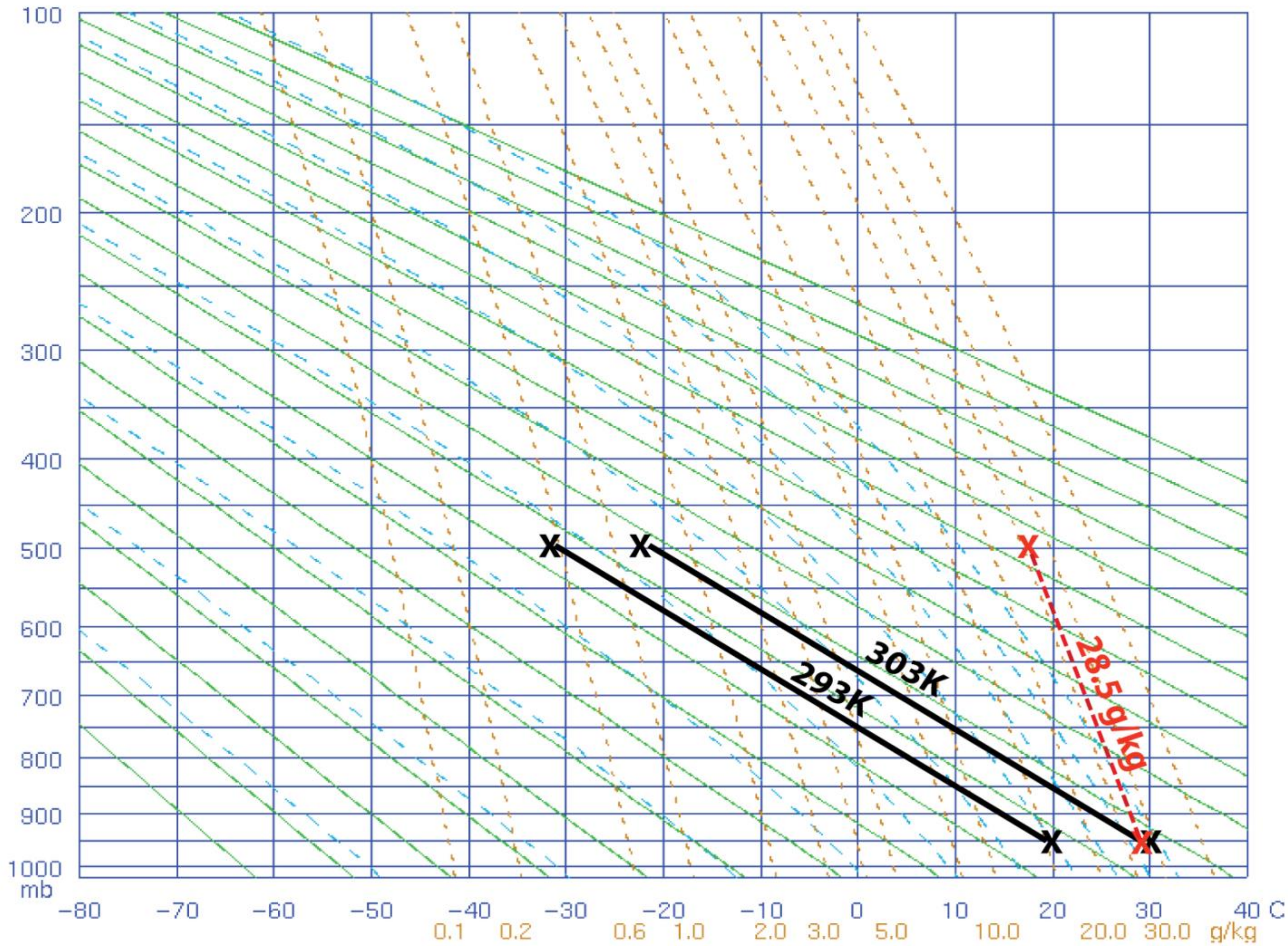
3. How does equation (3) suggest that saturation mixing ratio is directly proportional to temperature and inversely related to atmospheric pressure? (15 pts)

Equation (3) states that saturation mixing ratio is directly related to the numerator and inversely related to the denominator. The denominator is really the atmospheric pressure, since vapor pressure is two to three orders of magnitude smaller. Thus, the higher the atmospheric pressure the smaller the saturation mixing ratio.

The numerator is simply the saturation vapor pressure multiplied by a constant. But since saturation vapor pressure is directlly proportional to temperature, then saturation mixing ratio is directly related to temperature. At a given pressure, saturation mixing ratio is related to temperature only.

4. Comment on the general correspondence of your isohume and your isentropes to the isohumes and isentropes actually printed on the Stüve diagram? (15 pts)

Since the isentropes and isohumes on a thermodynamic diagram can be drawn on the basis of equations (1), (2), and (3), one would expect the lines we drew in this exercise should be parallel. That is what in fact was shown by this homework.


100
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0.1 0.2 0.6 1.0 2.0 3.0 5.0 10.0 20.0 30.0 g/kg
293K
303K
28.5 g/kg