Hypsometric Equation

The Ideal Gas Law is

$$p = \rho R_d T_v \qquad (1)$$

where the gas constant of dry air is retained by using the virtual temperature.

The Vertical Equation of Motion at the synoptic-scale is the Hydrostatic Equation (Law)

$$\frac{\Delta p}{\Delta z} = -\rho g \qquad (2)$$

Solving Equatioin 2 for density and inserting into equation (1), using calculus, eventually results in the following equation.

$$\Delta z = (z_2 - z_1) = \frac{R_d}{g} \cdot \overline{T_v} \cdot ln\left(\frac{p_1}{p_2}\right) \qquad (3)$$

which is known as the Hypsometric Relation (Equation). Since the pressure units cancel, it does not matter if the pressure is in kPa or mb.

The Hypsometric Relation allows one to relate the thickness of a slab of atmosphere bound by two isobars (say, the 1000 mb isobar and the 500 mb isobar) to the mean virtual temperature of the layer. Since, practically speaking, the difference between the mean virtual temperature and the actual temperature is very small, the Hypsometric Relation is often written

$$\Delta z = (z_2 - z_1) = \frac{R_d}{g} \cdot \bar{T} \cdot ln\left(\frac{p_1}{p_2}\right) \qquad (4)$$

The z in the above equations relate to geometric height above MSL. If Level 1 is taken at sealevel where z=0, then the thickness simply is the height of the given isobaric surface, e.g., the 200 mb height, or the 500 mb height.

In some applications, the geometric height is converted to a quantity known as geopotential height. This is because "g" in the equations above is the value at sea level. In reality, the value of g decreases with increasing distance from the center of the earth. At the 10 km level, there is about a 0.05% difference between the value of g there and the value of g at sea level. The calculations will result in an atmosphere that is slightly thicker than the use of the constant g would give. This difference is insignificant for the discussions we are having in this class, but becomes more significant of an issue in other applications.

To help you visualize the meaning of the equation, examine Figure 1.

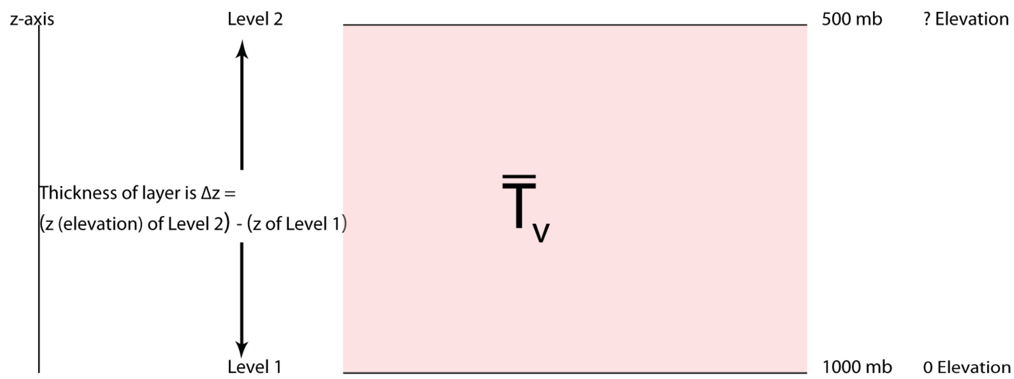

Figure 1: Schematic cross-section of a slab of atmosphere that extends from the ground where the pressure happens to be 1000 mb and the 500 mb level.

If the mean temperature of the atmosphere bounded by two pressure surfaces is known (by averaging temperatures on a sounding), then Equations (3) or (4) can be solved for the elevation of the pressure surface higher in elevation. Conversely, if the thickness of the atmosphere between two pressure surfaces is known (again, from the sounding information), then the mean temperature of the column of air bounded between them can be calculated.

For beginning students in meteorology, we often assume that Level 1 is at sealevel, and that the pressure at sealevel is 1000 mb. When we first begin looking at upper air weather maps, we often look at the 500 mb chart first. So, we'll make the assumption that Level 2 is the 500 mb level.

With these assumptions, here are the constants in Equation (4)

ln2=.693147; g=9.8 ms^{-2}; R=287.04 J kg^{-1} $K^{-1}$where a Joule is kg m^2 s^{-2}

which means the right hand side unit (when all cancellations are performed) is meters. Take a typical 1000-500 mb layer mean temperature of 277.9K = ~4C during the winter over Oakland. Insert this and the constants into Equation (4) gives a 1000-500 mb thickness (or 500 mb height if 1000 mb is the pressure at the MSL) of 564 dm = 5640 m. This is a realistic number for 500 mb heights.

But the most important conceptual "take away" from this derivation is simply this, the warmer the layer, the thicker (more vertically extensive) it is.