

Equation of Motion

1. *Newton's Second Law of Motion* -- states that the acceleration experienced by an object is due to the sum of the forces acting on the object . (An object at rest will be accelerated in proportion to the forces that act on the object). In meteorology, it is common to stratify this equation on the basis of horizontal and vertical accelerations, producing, essentially, two equations of motion.

$$\vec{a} = \sum \left(\frac{\vec{F}}{m} \right)$$

2. *Equation of Horizontal Motion (Navier-Stokes Equation)* -- air and water motion can be understood on the basis of the forces that cause air or water parcels to move

$$\vec{a}_h = \sum \frac{\vec{F}}{m} = \text{Pressure Gradient} \frac{\text{and}}{\text{or}} \text{Coriolis} \frac{\text{and}}{\text{or}} \text{Friction} \frac{\text{and}}{\text{or}} \text{Viscosity}$$

where subscript h means horizontal, either x,y or s, n. In meteorology this equation reduces to

$$\vec{a}_h = \sum \frac{\vec{F}_h}{m} = \text{Pressure Gradient} \frac{\text{and}}{\text{or}} \text{Coriolis} \frac{\text{and}}{\text{or}} \text{Friction}$$

3. *Equation of Vertical Motion* -- the upwards directed pressure gradient acceleration acting on an air parcel (explained in class) is balanced by the acceleration of gravity.

$$\vec{a}_z = \sum \frac{\vec{F}_z}{m} = \text{Pressure Gradient and Gravity and/or Buoyancy}$$

where $(PG)_z$ is the pressure gradient acceleration.

At the larger scales, buoyancy acceleration is zero. Most often net vertical accelerations produce vertical velocities that are two or three orders of magnitude

smaller than horizontal velocities and often can be neglected on an order of magnitude basis, because the vertical pressure gradient acceleration is balanced by the acceleration of gravity.

Hence

$$(PGA)_z = g$$

or

$$-\frac{1}{\rho} \frac{\Delta p}{\Delta z} = g$$

The Hydrostatic Law is often written:

$$\Delta p = -\rho g \Delta z$$