Calculating CAPE, Lifted Index and Strength
of Maximum Convective Updraft

CAPE Calculation
Lifted Index Calculation
Maximum Updraft Strength Calculation

I. Convective Available Potential Energy (CAPE)

A. Background

(i)   The vertical acceleration an air parcel experiences due to density differences **at a given level** is known as the buoyancy acceleration. This can be related to the difference in temperature of the air parcel, T_{ap}, with respect to the temperature of the surrounding air, T_e,

This can be derived from the gas law and the equation of vertical motion and is:

$$\overrightarrow{\left(\frac{\Delta w}{\Delta t}\right)} = \overrightarrow{a_z} = \frac{(T_{ap} - T_e)}{T_e}\vec{g}$$

Equation 1: Buoyancy Acceleration. Temperature in K.

Equation 1 states that the buoyancy acceleration can be determined from the difference in temperature between the lofted air parcel and the surrounding air AT THE SAME PRESSURE ELEVATION.

(ii)   A true measure of the *potential buoyancy* is a measure of the "positive" area on a Skew-T Ln P diagram. This represents the portion of the parcel ascent curve in which the parcel is warmer and, thus, less dense than the air surrounding it.

The parameter is known as Convective Available Potential Energy (CAPE) (or in some books and references, Positive Buoyancy (B+)).

$$CAPE = \left(\sum_{LFC}^{EL} \left[\frac{\left(T_{ap} - T_e\right)}{T_e} \vec{g} \right] Dz \right)$$

Note that this equation really states that CAPE is directly proportion to the total acceleration a parcel would experience due to buoyancy from the LFC to the EL.

Equation 2

The positive area represents a *potential* source of energy for parcels at the ground that are lifted to the elevation (LFC) above which they become warmer than their surroundings. To obtain this, one needs to algebraically add (that's what the summation sign means) this parameter at every level of the parcel's ascent until it reaches the point at which it becomes the same temperature as its surroundings again (Equilibrium Level).

Equation (2) states that CAPE is a function of the difference in temperature between the rising (or sinking) air parcel (T_{ap}) relative to the air around it at the same elevation (T_e). Note that the temperatures should be measured in Kelvin. CAPE is also a measure of the vertical acceleration that an air parcel experiences due to buoyancy summed up from the LFC to the EL.

CAPE is a measure of the area on a Skew T/Ln P diagram bounded by the curve of the ascending air parcel on the right and the environmental lapse rate (ELR) (sounding) on the left from the Level of Free Convection (LFC) (or, in the case of a forecast sounding for the afternoon based upon heating, the Convective Condensation Level c CCL) to the Equilibrium Level (EL). (Note: a similar expression is defined for Convective Inhibition Energy (CIN or CINH) in which the parcel curve lies to the left of the ELR.).

The summation sign in the equation means that the value of the expression in brackets needs to be evaluated at every level (and, of course, there are an infinite number of them) between the LFC (or CCL) and the EL. When you take calculus you will learn how to approximate the summation (called, finite differences) without making an extravagant number of calculations. But for the purposes of ERTH 260, you will simply divide the atmosphere

into 50 mb layers, from the LFC to the EL and calculate a "mean" CAPE for each of the layers and then sum those up to get the total or net CAPE.

B. Procedure

The procedure is relatively simple and is best done on a spreadsheet program (for example, Excel). Of course, if you are unfamiliar with spreadsheet programs, then using a hand calculator or doing it by hand will work (but is very tedious).

We will start with a schematic sounding (Fig. 1)--Remember. this is only an example, and does not correspont to a real case. Only the portion of the sounding from the LFC to the EL is shown. **Remember, the temperatures should be in Kelvin. So add 273⁰ to convert from Centigrade.**

The figure shows a schematic sounding with the following labels:

- 200 mb — EL — 12000 m
- 350 mb — 8500 m
- 500 mb — T_{e500} — T_{p500} — 6000 m
- $\Delta z = 1500$ m (between 6000 m and 4500 m)
- 650 mb — T_{e650} — T_{p650} — 4500 m
- 800 mb — LFC — 1500 m

CAPE for 650 mb - 500 layer is:

$$\left[\{[(T_{p500} - T_{e500})/T_{e500}] + [(T_{p650} - T_{e650})/T_{e650}]\}/2\right]g\Delta z$$

Note: $g = 9.8$ m/s^2, z in meters, T in Kelvin
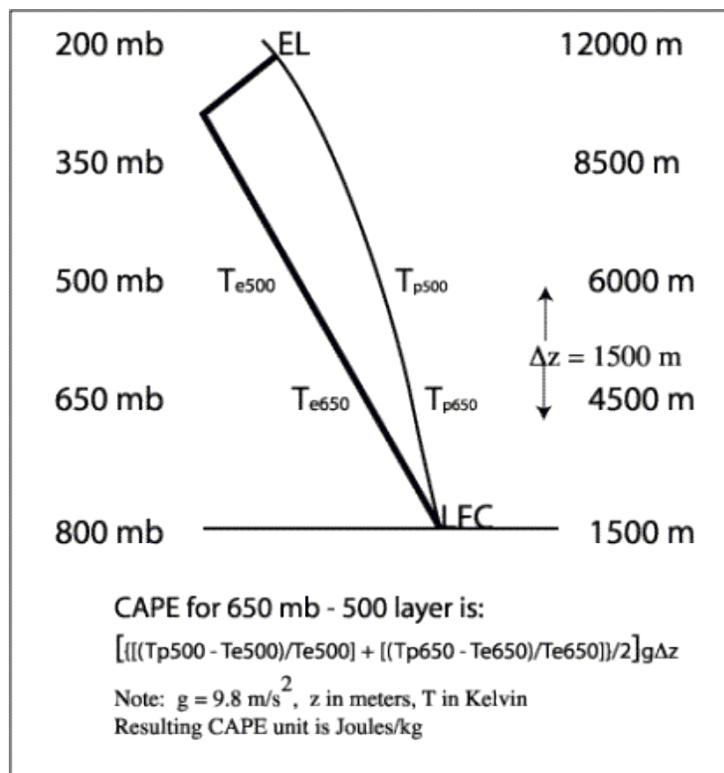Resulting CAPE unit is Joules/kg

Figure 1: Portion of schematic sounding from the LFC at 800 mb to the EL above the tropopause at 200 mb. Elevations of isobaric levels are shown. Procedure to estimate CAPE is summarized at bottom, as discussed in text.

The base of the portion of the sounding considered is indicated by the line at the 800 mb level. The thick solid line represents the ELR and the thin solid curve represents the wet adiabatic ascent curve of the rising air parcel. Note that the area of the polygon bounded by the two curves would be the CAPE.

To estimate the CAPE for a given layer, say the 650 mb to 500 mb layer, one would need to estimate the average value obtained from Equation 1 for the whole layer. A simple approximation of that would be to evaluate Equation 1 at each level , say the 500 mb level and at the 650 mb level, and just assume that the values calculated are representative of a layer centering on that level. Note that at the EL and the LFC, the procedure would yield zero CAPE.   Multiplying the result by g (9.8 m s^{-2}) and the thickness of the layer (in this case, 1500 m) would yield an estimate of the mean CAPE for the layer.

This procedure would need to be followed for each layer considered. In this example, there would be four values of layer CAPE estimates that you would need to add to estimate the total CAPE for the case.

In the case of this exercise, you will use layers 1500 m (1.5 km) thick. You'll be making the (bad) assumption that the conditions at each level represents conditions for a layer 1500 m thick centered at each level. Clearly, that is not estimating conditions correctly, but it is a start.

Procedure Detail Using a Real Sounding

We'll illustrate by using a real sounding, shown here.

Illustrating levels used in finite difference calculation of CAPE, as explained in class.

Step 1: After you have examined the sounding and determined the LFC (or CCL) and EL, you should tabulate the environmental temperature and air parcel temperature (remember, in Kelvin) for each pressure level from the LFC (or CCL) to the EL. Fill in the [Table below](#).

Pressure	Temp ap	Temp env	Temp Diff (Tap-Tenv)	Temp Quotient*
700				
600				
500				
400				
300				
240				

Table 1: Tally Table for CAPE Calculations

*Temperature Quotient = (Temp Diff/TempEnv)

Pressure	Elevation	Temp Parcel	Temp Env	Diff
700	3000	281	281	0
600	4500	273	269	4
500	6000	261	257.5	3.5
400	7500	253	248.5	4.5
300	9000	238	232	6
240	10500	219.5	219.5	0

Table 1a: Table 1 filled in for sounding example given above (I did this in Excel).

Step 2: Using the information from Table 1, you will need to compute the values needed to solve Equation 1 at each level. I suggest having a second table like the one given as Table 2.

	Delta z	gravity	Temp Quotient*	Layer CAP
	1500 m	9.8 m s-2		
	1500 m	9.8 m s-2		
	1500 m	9.8 m s-2		
	1500 m	9.8 m s-2		
	1500 m	9.8 m s-2		
	1500 m	9.8 m s-2		
			Total CAPE***	

Table 2: Computational Results (from Equation 1 and Table 1)

*Should really be Layer Average Quotient but what we will simply use is the values from Table 1

** Layer Cape = Layer Average Quotient X gravity X 1500 m

***Total CAPE = Add all the values above

Δz	g	Quotient	Layer CAPE
1500	9.8	0	
1500	9.8	0.014869888	218.5873606
1500	9.8	0.013592233	199.8058252
1500	9.8	0.018108652	266.1971831
1500	9.8	0.025862069	380.1724138
1500	9.8	0	
		Total CAPE	1064.762783

Table 2a: Table 2 filled in for sounding example given above. (I did this in Excel).

Once you have complete Table 2, you simply sum the values in the fifth column to obtain your estimate of CAPE for the sounding. It's going to be an underestimate because we used values for levels instead of layer averages. You can see how a computer could do this better, by computing layer averages for layers that were very small.

II. Lifted Index

The difference $[-(T_{ap} - T_e)]$ is called the LIFTED INDEX, commonly evaluated at the 500 mb level (negative for unstable conditions). You can calculate that from Table 1 for this particular case. Note that the algebraic negative sign in from of the expression means that the Lifted Index will return a negative value if the rising air parcel is warmer than its environment and vice versa.

For the sounding above, the LI is noted right on the sounding.

III. Convectively-Driven Vertical Velocity at the Equilibrium Level

The strength of the updraft will be greatest at the Equilibriium Level (EL). Thus, the greatest vertical velocities in cumulonimbus clouds are found at this level. Since, for each layer the air parcel traverses, it is given an additional acceleration (another bump up), the parcel will continue accelerating until the Equilibrium Level.

Above the Equilibrium Level the air parcel will have a negative (downward-directed) acceleration that will oppose the updraft. (In other words, the air parcel continues to rise because of its own upwards momentum, until the negative buoyancy due to the parcel finding itself colder than its surroundings produces a downward contribution of equal magnitude to the upward velocity.)

At that point, normally found about 1/4 to 1/3 the depth of the positive area on the sounding above the EL, the parcel will sink. The formula for the maximum upward motion (at the EL) due to buoyancy forces can be derived from the equations above. It is

$$w = [2 \text{ X CAPE}]^{1/2}$$

where w is the vertical velocity in the Cartesian Cooridinate system (x, y, z). For the homework problem, all you need to do is to multiply the CAPE value you got in Table 2 by 2 and then take the square root. Before you do that, make sure that the units work out.

For the sounding above,

$w = [2 \text{ X } 1064 \text{ J/kg}]^{1/2} = 46.1 \text{ m s}^{-1}$