

Divergence Related to Gradient Wind

The equation of motion in natural coordinates is

$$kV^2 = -fV - g\left(\frac{\partial z}{\partial n}\right) \quad (1)$$

where k is the trajectory curvature.

The relationship between the curvature of a trajectory and the curvature of a streamline is given by the so-called “Blaton’s Formula”

$$k = k_s \left(\frac{V - c}{V} \right) \quad (2)$$

where k_s is the streamline curvature, V is the wind speed and c is the motion of the moving trough or ridge.

Put (2) into (1) and solve for V , now given a subscript to indicate that this is now the gradient wind.

$$V_{gr} = -\frac{g}{f} \frac{\partial z}{\partial n} - \frac{k_s}{f} (V - c)V$$

or, with substitution of the definition
for the geostrophic wind

$$V_{gr} = V_g - \frac{k_s}{f} (V - c)V$$

(3a,b)

This says that the gradient wind differs from the geostrophic wind by a term proportional to the streamline curvature. What are the implications of this? Fool around with the equation by considering its implication in different portions of the troposphere.

Inclass Exercise 5: Gradient Wind Divergence

1. Examine Equation (3b). Discuss how the equation mathematically suggests that flow around upper tropospheric ridges should be supergeostrophic (faster than that predicted by the geostrophic wind relation) and flow around troughs should be subgeostrophic.
2. Consider (see graphic below): (a) a low amplitude short wave trough ridge system in the upper troposphere in which the height gradient is everywhere constant;; and, (b) a low amplitude, but high wavelength trough ridge system in the upper troposphere in which the height gradient is everywhere constant. Sketch each pattern, and place a vector for the geostrophic wind and the gradient wind at trough and ridge axis.

The kinematic definition of horizontal divergence in natural coordinates for patterns in which the height contours are absolutely parallel to one another is $\partial V / \partial s$.

Qualitatively assess the divergence at the inflection point for each case.

