Converting Horizontal Pressure Gradient to a Height Gradient

$$a_{p_x} = -\frac{1}{\rho}\frac{\Delta p}{\Delta x} \qquad (1)$$

The hydrostatic law is

$$\Delta p = -\rho g \Delta z \qquad (2a)$$

$$\rho = -\frac{1}{g}\frac{\Delta p}{\Delta z} \qquad (2b)$$

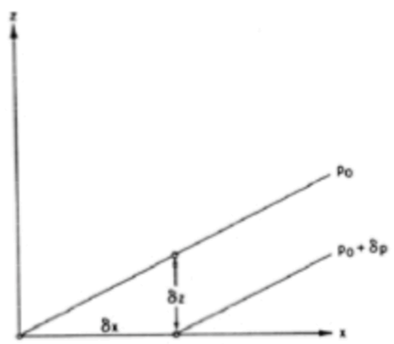

Figure 1: A view of isobars in cross section, in which pressure decreases upward

$\Delta p = p_2 - p_1$

$On\ x - axis,\ p_2 - p_1 = (p_0 + \delta p) - p_0$

$On\ z - axis,\ p_2 - p_1 = p_0 - (p_0 + \delta p)$

where δp is the interval between isobars.

Hence, Δp on x-axis = -Δp on z-axis

or $(\Delta p)_x = -(\Delta p)_z$ (3)

Put (3) into (2b)

$$\rho = \frac{1}{g}\frac{(\Delta p)_x}{\Delta z} \qquad (4)$$

Put (4) into (1)

$$a_{p_x} = -\frac{1}{\rho}\frac{\Delta p}{\Delta x} = -g\frac{\Delta z}{\Delta x} \qquad (5a, b)$$

$$a_{p_x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial z}{\partial x}$$

A comparable version of (5b) can be obtained for the y component of the pressure gradient acceleration.

$$a_{p_y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} = -g\frac{\partial z}{\partial y} \qquad (6)$$

the quantity g∂z is also known as geopotential height and is given the symbol ϕ.

Hence, the horizontal pressure gradient accelerations can be expressed as

$$a_{p_x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial z}{\partial x} = -\frac{\partial \Phi}{\partial x} \qquad (7a)$$

$$a_{p_y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} = -g\frac{\partial z}{\partial y} = -\frac{\partial \Phi}{\partial y} \qquad (7b)$$

and in the natural coordinate system

$$a_{p_s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} = -g\frac{\partial z}{\partial s} = -\frac{\partial \Phi}{\partial s} \qquad (8a)$$

$$a_{p_n} = -\frac{1}{\rho}\frac{\partial p}{\partial n} = -g\frac{\partial z}{\partial n} = -\frac{\partial \Phi}{\partial n} \qquad (8b)$$