Coriolis Acceleration in Rectangular Coordinates

1. General

When discussing acceleration relative to the earth due to Coriolis effect, I give the following table to my students.

Coriolis Acceleration	
1.	Deflects all frictionless moving objects to the right of their motion in the Northern Hemisphere (except at Equator, see (2) below).
2.	Deflection is maximum at Poles and zero at Equator.
3.	Deflection is greater the faster an object is moving in a given time interval.
4.	Deflection is greater the longer the time an object is in motion at a constant speed.

I have been careful to go over this because some of the students have taken Toby Garfield's physical oceanography class. In discussing Coriolis' impact on ocean currents, the oceanographers often mention that despite the relatively slow motion of the ocean currents, Coriolis effects are great. This led to confusion since in the traditional meteorological treatment of Coriolis we usually only relate the magnitude of the acceleration to the latitude and the speed of motion.

2. Nature of the Deflection

The west-east acceleration experienced by a meridional component is

$$\left(\frac{du}{dt}\right)_{Coriolis} = 2\Omega v \sin\phi \qquad \text{(1a)}$$

$$\left(\frac{dv}{dt}\right)_{Coriolis} = -2\Omega u \sin\phi \qquad (1b)$$

If one assumes that an air parcel moves at a constant speed at given latitude, then (1a) can be integrated twice with respect to time to get the magnitude, x, of the horizontal deflection due to Coriolis

$$\mathbf{dx}/\mathbf{dt} = 2\Omega\mathbf{v}\sin\phi\mathbf{t} \qquad (2)$$

$$\mathbf{x} = \Omega\mathbf{v}\sin\phi\mathbf{t}^2 \qquad (3)$$

The deflections obtained from (3) for various wind speeds and time intervals are given below. The justification for the oceanographer's observation that "...something moving slow can be deflected more than something moving fast..." can be deduced from the last two items in the table.

x (meters)	v (m s^{-1})	t (sec)
6074	10	3600
12149	20	3600
0.02	100	2
0.94	2	100

3. Assessing the Validity of the Geostrophic Wind Approximation

The Rossby Number is a dimensionless ratio that assesses the degree to which the geostrophic assumption is valid. It is the ratio of the total acceleration to the Coriolis acceleration as given in the equation of horizontal motion (the horizontal momentum equation), applied in the middle troposphere..

$$R_o = \frac{\left| \frac{du}{dt} \right|}{\left| 2\Omega \sin\phi v \right|} \qquad (4)$$

For geostrophic flow, the Rossby Number vanishes and is undefined at the Equator (why?). The Rossby Number is >2.0 in the lower subtropics. In the middle latitudes the Rossby Number is less than 0.1 on average. Values of the Rossby Number in the middle latitudes are <0.05 in the mid troposphere. Thus, in general, the geostrophic wind approximation works well in explaining the actual wind field in the middle latitudes.