

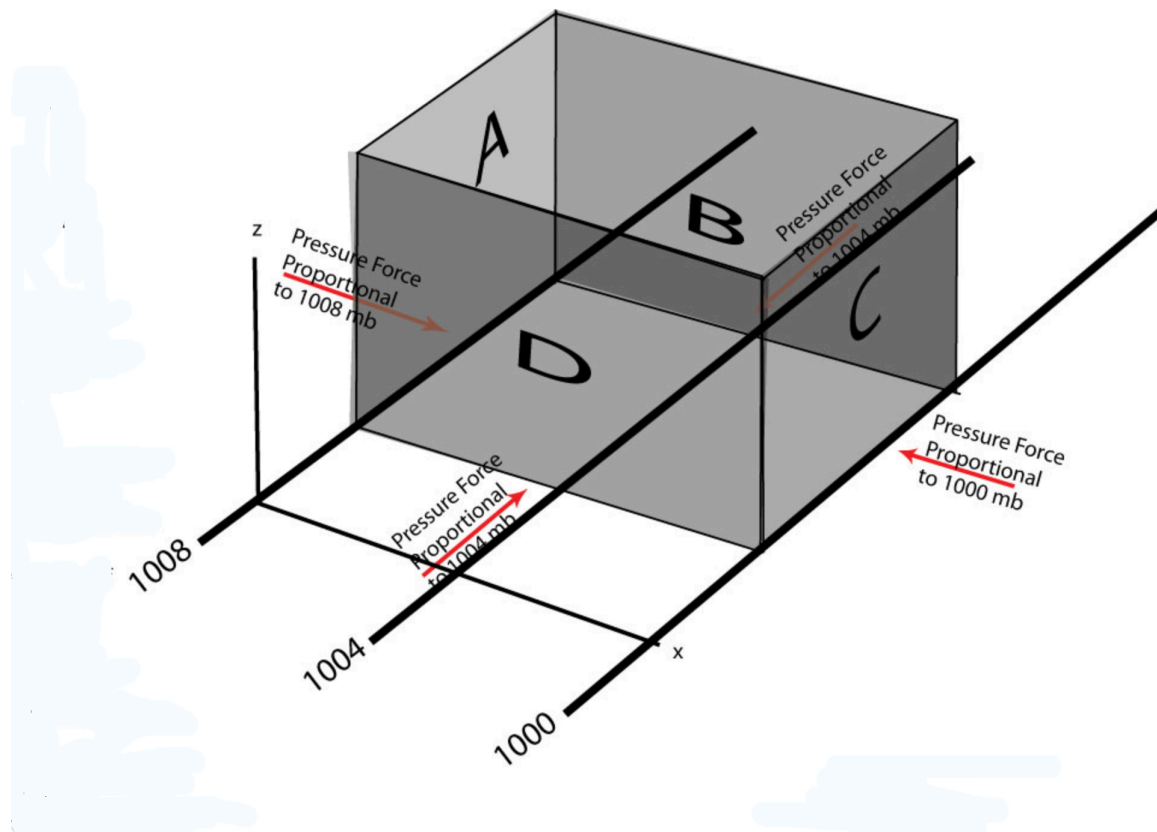
Pressure Force and the Pressure Gradient Acceleration

The consideration of the forces that influence atmospheric flow is key in weather forecasting. Force is the quantitative description of the interaction between two physical bodies, and was fully defined by Sir Isaac Newton in his Laws of Motion.

In the case of the example (illustrating the force of pressure) discussed in class, pressure is defined as a force per unit area, or

$$p = F_p/A \text{ or } F_p = pA \quad (1)$$

where A is the area over which the pressure force acts.



The net pressure force across a cube-shaped air parcel is the algebraic sum of the pressure forces, F_p , that act across all the faces of the air parcel. These forces are vectors that have magnitude and direction, and can act either in the positive or negative direction on the coordinate axes. The term “net” implies an algebraic addition of the vectors (in this case, forces) that act along a given axis.

For example, using the rectangular coordinate system, the net force along the x-axis acts on the face of the air parcel with dimensions on the y and z axes, or $A = \Delta y \Delta z$

$$\text{net } \vec{F}_{p_x} = [(p_C + p_A) \Delta y \Delta z] \vec{i}$$

but p_C acts in the negative direction at location 2, $p_C = -p_2$

and p_A acts in the positive direction at location 1, $p_A = p_1$ (2)

So that the net pressure force is

$$\text{net } \vec{F}_{p_x} = -[(p_2 - p_1) \Delta y \Delta z] \vec{i}$$

(3)

Newton's Second Law of Motion states that the acceleration experienced by an object with mass m is proportional to the sum of the forces acting on the object.

$$\vec{a} = \vec{F} / m \quad (4)$$

which states that the acceleration can be determined by adding up the forces (net force) that act on an object of mass m

Substituting (3) into (4) gives an expression for the pressure gradient acceleration acting along the x-axis.

$$\vec{a}_{p_x} = \frac{-[(p_2 - p_1)\Delta y \Delta z]}{m} \hat{i} \quad (5)$$

But mass is density X volume or

$$m = \rho \Delta x \Delta y \Delta z \quad (6)$$

Substituting (6) into (5) gives

$$\vec{a}_{p_x} = \frac{-[(p_2 - p_1)]}{\rho \Delta x} \vec{i} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} \vec{i} \quad (7)$$

where the definition of the finite difference approximation is used on the far right hand term.

Similar arguments can be made for the net pressure forces acting on the y and z axes to obtain the three dimensional pressure gradient acceleration.

$$\vec{a}_p = \frac{\vec{F}_p}{m} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} \vec{i} - \frac{1}{\rho} \frac{\Delta p}{\Delta y} \vec{j} - \frac{1}{\rho} \frac{\Delta p}{\Delta z} \vec{k} \quad (8a)$$

and converting to calculus notation

$$\vec{a}_p = \frac{\vec{F}_p}{m} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \vec{i} - \frac{1}{\rho} \frac{\partial p}{\partial y} \vec{j} - \frac{1}{\rho} \frac{\partial p}{\partial z} \vec{k} \quad (9b)$$

and then to vector calculus shorthand

$$\vec{a}_p = \frac{\vec{F}_p}{m} = -\frac{1}{\rho} \nabla p \quad (10c)$$