

Inclass Exercise #2 Key: Using the Del-Operator---  
Three-dimensional Divergence of the Wind (25 pts)

The three dimensional wind vector in Cartesian coordinates is

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \quad (1)$$

The Del operator is

$$\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \quad (2)$$

A mathematical expression for the three dimensional divergence of the wind is given by the dot product of the del operator with the wind.

$$\nabla \cdot \vec{V} = (\text{Divergence of the Wind})_{3d} \quad (3)$$

Inclass Exercise

Perform the dot product operation implied by (3) using equations (2) and (1). Remember that the del-operator is a derivative and the dot product of it with a vector is essentially giving the derivatives of the dependent variable along each of the three coordinate axes.

$$\nabla \cdot \vec{V} = \left( \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \right) (u\vec{i} + v\vec{j} + w\vec{k})$$

$$\nabla \cdot \vec{V} = \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$