San Francisco State University Name _____________
Department of Earth & Climate Sciences Fall 2017

ERTH 465
Inclass Exercse 5: Divergence Related to Gradient Wind
Due Thursday 28 September (50 pts)

The so-called Gradient Wind Relation in Natural Coordinates is

$$V_{gr} = -\frac{g}{f}\frac{\partial z}{\partial n} - \frac{k_s}{f}(V-c)V$$

or, with substitution of the definition for the geostrophic wind

$$V_{gr} = V_g - \frac{k_s}{f}(V-c)V \qquad \text{(3a,b)}$$

This says that the gradient wind differs from the geostrophic wind by a term proportional to the streamline curvature. What are the implications of this? Fool around with the equation by considering its implication in different portions of the troposphere.

1. Examine Equation (3b). Discuss how the equation mathematically suggests that flow around upper tropospheric ridges should be supergeostrophic (faster than that predicted by the geostrophic wind relation) and flow around troughs should be subgeostrophic.

Since anticyclonic curvature is negative, flow around ridges will produce a centrifugal accerlation "correction term" that adds to the geostrophic wind value estimated for the given contour spacing. Thus, the wind speed at ridge axes should be greater than that estimated for the geostrophic wind alone (supergeostrophic).

Since cyclonic curfacture is positive, flow around troughs, say, at trough axes, will produce a centrifugal acceleration correction term that subtracts from the geostrophic wind value estimated for the given contour spacing. Thus, the wind speed at the trough axes should be smaller than that estimated for the geostrophic wind alone (subgeostrophic).

2. Consider (see graphic below): (a) a low amplitude short wave trough ridge system in the upper troposphere in which the height gradient is everywhere constant;; and, (b) a low amplitude, but high wavelength trough ridge system in the upper troposphere in which the height gradient is everywhere constant. Sketch each pattern, and place a vector for the geostrophic wind and the gradient wind at trough and ridge axis.

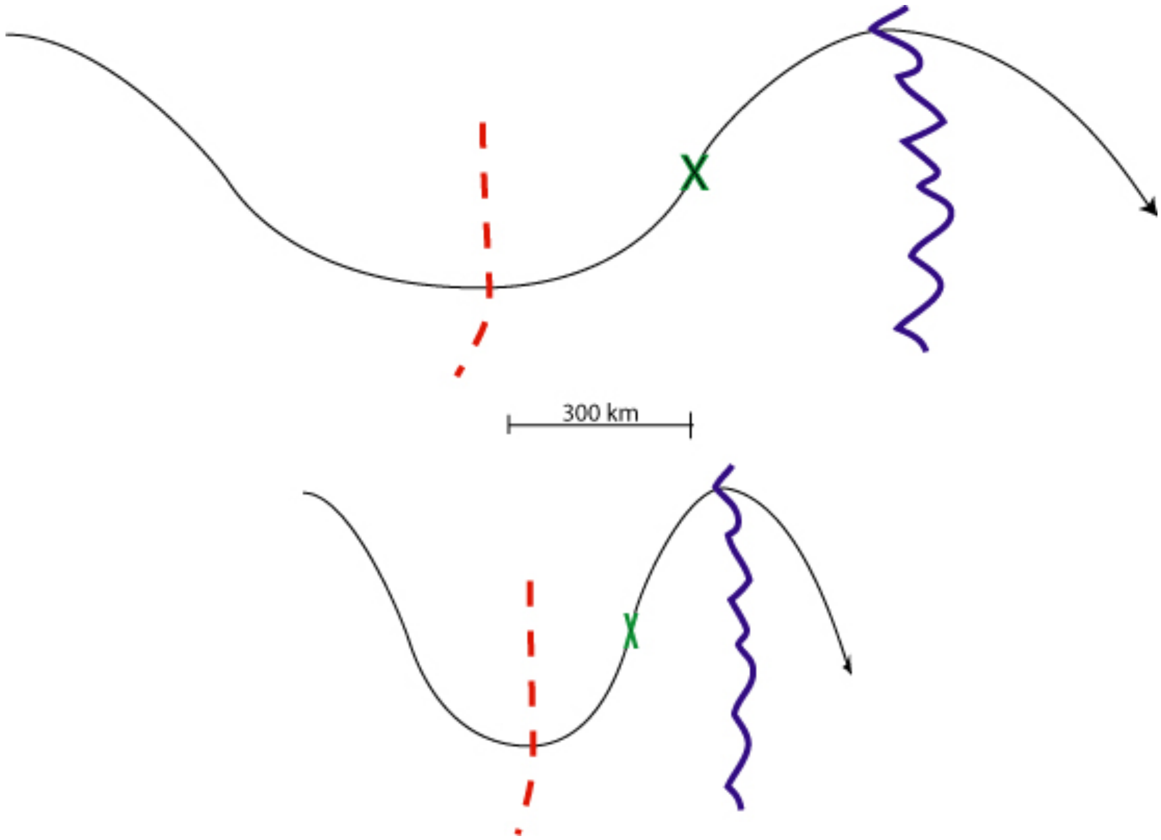
The kinematic definition of horizontal divergence in natural coordinates for patterns in which the height contours are absolutely parallel to one another is $\partial V/\partial s$.

Qualitatively assess the divergence at the inflection point for each case.

All else being equal, the absolute value of the centrifugal acceleration "correction term" will have the greatest magnitude for larger curvatures. Thus, short waves will have winds that are the most subgeostrophic at trough axes and most supergeostrophic at ridge axes.

Since horizontal divergence (the way it is defined here: $\partial V/\partial s \sim \Delta V/\Delta s$) is proportional to the difference in winds between the ridge axis (V_2) and trough axis (V_1), that difference will be the largest for short waves and smallest for long waves. (Note: I realize that the Δs is larger for long waves, but it turns out the numerator of this equation dominates typically.)

Short waves have greater divergence/convergence associated with them than long waves. And long waves tend to be nearly non-divergent.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