More on Temperature Advection and

Some Vector Calculus Notation

$$-\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)= \text{ Temperature Advection}_{\text{Cartesian}}$$

$$-\left(V\frac{\partial T}{\partial s}+w\frac{\partial T}{\partial z}\right)= \text{ Temperature Advection}_{\text{Natural}}$$

$$-\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)= \text{ Horizontal Temperature Advection}_{\text{Cartesian}}$$

$$-\left(V\frac{\partial T}{\partial s}\right)= \text{ Horizontal Temperature Advection}_{\text{Natural}}$$

Note: In order for temperature to be advected, there must be a temperature difference (gradient) along the streamline. Hence, the temperature gradient normal to the streamline $\Delta T/\Delta n$ does not contribute to temperature advection, since the wind, by definition, would be parallel to the isotherms.

$$\nabla = \vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y + \vec{k}\,\partial/\partial z$$

Del Operator. Is a short hand way o indicating a three-dimensional gradient. $\vec{i}$, $\vec{j}$, and $\vec{k}$ are so-called "unit" vectors (corresponding to an increment of 1, on the x, y and z axes.)

$$\nabla_h = \vec{i}\,\partial/\partial x + \vec{j}\,\partial/\partial y$$

Horizontal Del Operator. Has x and y component. Multiply by a scalar just gives the spatial derivatives of the scalar.

$$\nabla_h T = \vec{i}\,\partial T/\partial x + \vec{j}\,\partial T/\partial y$$

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$$

Three Dimensional Wind Vector

Dot product: Multiply each cartesian component of a vector by the cartesian component of the other vector.

For example,

$$(a\vec{i} + b\vec{j} + c\vec{k}) \bullet (d\vec{i} + e\vec{j} + f\vec{k}) = ab + be + cf$$

Note: unit vectors dotted with themselves return a value of 1; unit vectors dotted with a different unit vector return a value of 0.

Thus

$$-\vec{V} \bullet \nabla_h T = -\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

Note that although the wind vector has a k component, the horizontal del operator does not. Hence, the k product returns a zero. The equation above is the vector calculus version of horizontal temperature advection.

Advection is defined as the product of the wind and the gradient of some atmospheric variable. Even the advection of the wind by itself (the product of the wind vector with the gradient of the wind).

Erratum: Correction of expression above.

$$\left(a\vec{\imath} + b\vec{\jmath} + c\vec{k}\right) \cdot \left(d\vec{\imath} + e\vec{\jmath} + f\vec{k}\right) = ad + be + cf$$