Relationship between the vertical velocity
in x,y,z and that in x,y,p

The vertical velocity in x,y,p coordinates is the rate at which a vertically-moving air parcel experiences a change in pressure following its motion. Recall the Lagrangian derivative of pressure is:

$$Dp/Dt = \partial p/\partial t + V\left(\partial p/\partial s\right) + w\left(\partial p/\partial z\right) \qquad (1)$$

What are the typical values of each of the terms to the right of the equals sign? Surface pressure tendencies are on the order of 1 to 4 mb per hour. The pressure change experienced by an air parcel moving, say, 10 m/s over one hour would be, even in a region of strong pressure gradient, around 2 mb per hour. The pressure change experienced by an air parcel rising with a vertical velocity of 10 cm/s (360 m per hour) would be around 100 mb. Thus the last term is two orders of magnitude larger than the other terms.

The total pressure change is mostly due to vertical motion and then is written

$$Dp/Dt \approx \omega = w\left(\partial p/\partial z\right) \qquad (2)$$

where the lower case Greek letter omega is used as a symbol for the pressure change experienced by a vertically moving air parcel. Since, at the synoptic scale, most of the pressure change experienced by moving air parcels occurs as they move vertically, the change in pressure experienced by these parcels will be a function of their vertical motion relative to the vertical pressure gradient. This is defined as the vertical velocity in the x, y, p coordinate system, or ω. Upward motion occurs for negative values of ω.

Substituting the hydrostatic approximation into the right hand side of (2) gives the following definition of the vertical velocity in x,y,p coordinates:

$$\omega = -\rho g w \qquad (3)$$

Equation (3) says that the vertical velocity in the two coordinate systems differ by a factor proportional to (density X gravity). For a typical surface density of 1.0 X 10^{-3} g/cm^{-3} and gravity of 9.8 X 10^{2} cm s^{-1}, at the synoptic scale we get vertical velocities on the order of 10 microbars per second and 10 centimeters per second (roughly).