The Thermal Wind

The geostrophic wind in natural coordinates (in this case, s, n, p coordinates) is

$$V_g = -\frac{g}{f}\left(\frac{\partial z}{\partial n}\right) \qquad (1)$$

where **∂z/∂n** is the height gradient normal to the streamlines.

Suppose you were on a desert island, and only had access to the surface (1000 mb) geostrophic wind information and surface temperature field in your vicinity. Is it possible to estimate the 500 mb geostrophic wind just based upon that? The answer is yes.

First, let's consider the difference between the 500 mb geostrophic wind and the 1000 mb geostrophic wind. Equation (2) gives the difference between the 500 mb and 1000 mb geostrophic winds.

$$V_{g_{500}} - V_{g_{1000}} = \left[-\frac{g}{f}\left(\frac{\partial z}{\partial n}\right)\right]_{500} - \left[-\frac{g}{f}\left(\frac{\partial z}{\partial n}\right)\right]_{1000} \qquad (2)$$

If the increment along the n-axis is the same, equation (2) can be rewritten as follows:

$$V_{g_{500}} - V_{g_{1000}} = \left[-\frac{g}{f}\left(\frac{\partial z_{500} - \partial z_{1000}}{\partial n}\right)\right] \qquad (3)$$

But $z_{500} - z_{1000}$ is just the 1000-500 mb thickness $Z_{1000\text{-}500}$.

Thus, equation (3) can be rewritten as:

$$V_{g_{500}} - V_{g_{1000}} = \left[-\frac{g}{f}\left(\frac{\partial Z_{1000-500}}{\partial n}\right)\right] \qquad (4)$$

Note that the right hand side of (4) has the appearance of a geostrophic wind. In fact, the right hand side is known as the "Thermal Wind", V_t.

Equation (4) can be rewritten

$$V_{g_{500}} - V_{g_{1000}} = V_t \qquad (5a)$$

or

$$V_{g_{500}} = V_t + V_{g_{1000}} \tag{5b}$$

The concept that emerges is that the thermal wind is a correction factor which, when added to the lower level (say, surface) geostrophic wind field can produce an estimate of the geostrophic wind field aloft. Since the wind field at 500 mb is most nearly geostrophic then this procedure can give you a qualitative estimate of the mid-tropospheric wind field.

Just as the geostrophic wind in the Northern Hemisphere blows parallel to the height contours and counter clockwise (clockwise) around areas of low (high) values, the Thermal Wind blows parallel to the thickness contours and counter clockwise (clockwise) around areas of low (high) values.

Thought Question: The 1000-500 mb thickness contours are zonally oriented and the surface wind is calm. What is your estimate for the 500 mb wind direction? Use equation (6) and describe what assumptions you made.

The Thermal Wind in finite difference form is

$$V_t = -\,{}^{g}\!/\!{}_{f}\left(\frac{Z_2 - Z_1}{\Delta n}\right) \tag{6}$$

where Z is the thickness.

The hypsometric equation states that the thickness of a layer bounded by two isobars is directly proportional to its mean virtual temperature.

$$Z = \mathbf{R/g\ \ ln\ p_1/p_2\ \overline{T}_v} \qquad (7)$$

For our purposes, the virtual temperature can be assumed to be the same as the temperature. Equation (7) should be substituted into (6) in the following manner:

$$Z_2 = \mathbf{R/g\ \ ln\ 2T_2} \qquad (8a)$$

$$Z_1 = \mathbf{R/g\ \ ln\ 2T_1} \qquad (8b)$$

where T_2 and T_1 are the mean temperatures of the layer bounded by the 1000 and 500 mb isobars at locations 2 and 1, respectively. Inserting (8a) and (8b) into (6) gives

$$V_t = -\left({}^{R}\!/_{f} \right) \ln 2 \left(\frac{T_2 - T_1}{\Delta n} \right) \qquad (9a)$$

$$V_t = -\left({}^{R}\!/_{f} \right) \ln 2 \left(\frac{\partial T}{\partial n} \right) \qquad (9b)$$

which states, simply, that thermal wind at a given spot is directly proportional to the mean temperature gradient normal to the flow. If one makes the assumption that deep temperature gradients (for example, across fronts) are consistent through the troposphere, equation (10a,b) allows one to answer a question like this: What is the 500 mb wind vector if the surface winds are calm and temperatures decrease northward at 5º per 500 km?