Dot Product Involving the Three Dimensional Wind Vector

A. Representing the Wind as a Vector

The three dimensional wind vector in rectangular coordinates is

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \qquad (1)$$

where i, j and k are the unit vectors along the x, y and z coordinate axes, respectively. Please note that in some books the three dimensional wind vector is given as v. In some books, the symbol for the three dimensional velocity vector is given as U to prevent confusion with the wind component in the natural coordinate system.

The **i, j,** and **k** represent unit vectors with magnitude 1 on each of the coordinate axes, while the quantity multiplied by the unit vector is just a magnitude. Thus, one can have u**i,** which simply means that the u component of the wind has a magnitude but a direction along the x-axis. The "unit" is always defined as a magnitude of 1 in the corresponding unit-system.

By the way, sometimes equation (1) can be written as

$$\vec{V} = \vec{V}_h + w\vec{k}$$

where

$\vec{V}_h$ is the horizontal wind vector and

$$\vec{V}_h = u\vec{i} + v\vec{j}$$

B. The Dot Product

The dot product is a vector algebra operation that resembles a multiplication using the rule of the product, but whose result actually determines the component each vector has upon the other.

The procedure is simple. Multiply the x-component of each vector to the x-component of the other, etc. Since the dot product of a unit vector with itself yields 1, then the procedure becomes simpler.

For example suppose there is a dot product of a vector

$$\vec{A} = a\vec{i} + b\vec{j} + c\vec{k} \tag{2}$$

with the three dimensional wind vector as given in equation (1).

$$\vec{V} \bullet \vec{A} = \left(u\vec{i} + v\vec{j} + w\vec{k}\right) \bullet \left(a\vec{i} + b\vec{j} + c\vec{k}\right)$$

$$\vec{V} \bullet \vec{A} = \left(ua + vb + wc\right)$$

The dot product of two vectors always results in a scalar. You can see how it works. If there is a dot product of two vectors the result will have no unit vectors. The answer is the sum of the products of the components of each vector.