Negatively Tilted, High Amplitude Short Wave in Middle Latitudes

Term A	Large Positive
Term B	Large Negative
Term C	Very Large Positive
Term D	Large Positive
Term E	Large Positive

Net Effect-- Divergence

If one obtains the so-called critical speed $V_c=V_L-c=\beta L^2/4\pi^2$ from (3) and substitutes that into (4) one obtains the following:

$$\bar{D} = \frac{16\pi^2 A}{\bar{f} L^3} V(V - V_L) \qquad (5)$$

The magnitude of the gradient wind divergence is greatest if the wave amplitude is large, the wavelength is small, and the wind speed is large and significantly different from that at the level of nondivergence.

Appendix 1

<u>Case 1</u>

Positively Tilted, High Amplitude Short Wave in Middle Latitudes

Term A	Large Positive
Term B	Large Negative
Term C	Very Large Positive
Term D	Large Positive
Term E	Negative

Net Effect--Strong Divergence

<u>Case 2</u>

Neutrally Tilted, High Amplitude Long Wave in Middle Latitudes

Term A	Small Positive
Term B	Very Large Negative
Term C	Small Positive
Term D	Large Positive
Term E	Zero
Net Effect	Weak Convergence

<u>Case 3</u>

Divergence east of a trough axis is therefore GREATEST

1. the higher the wind speed;
2. the larger the amplitude of the wave;
3. the greater the speed of the wind relative to the wave;
4. the smaller the wavelength;
5. the higher the latitude.

At the level of nondivergence, Equation (2) reduces to Rossby's Wave Speed Equation.

$$\mathbf{c} = \mathbf{V_L} - \frac{\beta \mathbf{L}^2}{4\pi^2} \quad (3)$$

(Note: V_L is the non-divergent wind, or, the geostrophic wind; or the wind at the level of non-divergence).

B. <u>More About Baroclinic Waves</u>

Substitution of equation (3) (after isolating the wave speed c and the wind speed at the level of non-divergence V_L on right hand side) into (2) gives:

$$\bar{\mathbf{D}} = \frac{4\mathbf{VA}\beta}{\bar{\mathbf{f}}\,\mathbf{L}} \left\{ \frac{\mathbf{V} - \mathbf{c}}{\mathbf{V_L} - \mathbf{c}} - 1 \right\} \quad (4)$$

Assuming that phase speed of a baroclinic wave at all levels is the same, note that where the wind speed differs from the speed at the LND the term in bracket returns either a positive or negative value. For example, at the 300 mb level $V >> V_L$ and the waves are strongly divergent east of the trough axes. Beneath the LND the opposite is true. This is another application of Dine's Compensation. At the LND, $V=V_L$ and the wave is nondivergent.

Divergence and Upper Waves

A. Divergence East of Trough Axis in Sinuosoidal Patterns

Rossby's Wave Speed Equation can be derived in another manner. Using the fundamental definition of divergence and the definition of the gradient wind, the divergence that occurs from trough line to downstream ridge line for a sinusoidal wave can be derived. The expression is:

Term A Term B Term C

$$\bar{\mathbf{D}} = \frac{4\mathbf{V}}{\mathbf{L}}\left\{\frac{(\mathbf{f_t} - \mathbf{f_r}) + \mathbf{V}(\mathbf{K_t} - \mathbf{K_r})}{(\mathbf{f_t} + \mathbf{f_r}) + \mathbf{V}(\mathbf{K_t} + \mathbf{K_r})}\right\} \qquad (1)$$

Term D Term E

where the subscripts t and r refer to trough and ridge axis, respectively, K is the trajectory curvature and V is the mean wind speed.

Note that $(f_t\text{-}f_r)$=Term B is large negative the greater the amplitude, $(K_t\text{-}K_r)$=Term C is large the greater the variation of curvature between trough and ridge (generally inversely related to wavelength), (f_t+f_r)=Term D is large the higher the latitude (because f is greatest the higher the latitude) and (K_t+K_r)=Term E only returns a value if there is a difference in the absolute value of the curvature between trough and ridge (generally occurs if the trough and ridges are positively or negatively tilted). Term A is 4V/L which is small for long waves and large for short waves and always positive. (See Appendix 1)

Substitution of the relationship between STREAMLINE and TRAJECTORY curvature, and letting the streamlines be functions approximated by $y=A\sin 2\pi[(x-ct)/L]$ and utilizing the definition for ß gives:

$$\bar{\mathbf{D}} = \frac{4\mathbf{VA}}{\bar{\mathbf{f}}\,\mathbf{L}}\left\{(\mathbf{V} - \mathbf{c})\frac{4\pi^2}{\mathbf{L}^2} - \beta\right\} \qquad (2)$$