L can be solved for the “critical wavelength”. Wavelengths larger than this will be associated with waves that retrogress.

$$y = R\phi$$
$$\partial y = R\partial\phi \qquad \text{(ii)}$$

where R is radius of earth

$$\partial(2\Omega \sin\phi) = 2\Omega \cos\phi \partial\phi \qquad \text{(iii)}$$

Put (ii) and (iii) into (i)

$$\frac{\partial f}{\partial y} = \beta = \frac{2\Omega \cos\phi}{R} \qquad \text{(iv)}$$

put (iv) into (13a)

$$c = u - \frac{(\Omega \cos\phi)L^2}{2R\pi^2} \qquad \text{(13b)}$$

The equation states that the phase speed of a wave is directly related to wind speed modified by effects due to the wavelength (and latitude). For a given wavelength, the faster the zonal wind speed, the faster the motion of the waves.

For a given zonal wind speed, short waves will progress more faster than long waves. The critical speed is defined as that value of zonal wind speed in which waves of a given wavelength will become stationary. In other words,

$$c = 0$$

$$if$$

$$u = \frac{\beta L^2}{4\pi^2}$$

$$u_{critical} = \frac{\beta L^2}{4\pi^2} \qquad \text{(14)}$$

$$\frac{\partial \zeta}{\partial \mathbf{t}} \approx -\mathbf{c}\frac{\partial \zeta}{\partial \mathbf{x}} \qquad (6)$$

By assumption 2 above

$$\mathbf{w}\frac{\partial \zeta}{\partial \mathbf{z}} \approx 0 \qquad (7)$$

Put (5), (6), and (7) into (4)

$$(\mathbf{u} - \mathbf{c})\frac{\partial \zeta}{\partial \mathbf{x}} = -\mathbf{v}\beta \qquad (8)$$

The definition of relative vorticity is

$$\zeta = \left(\partial \mathbf{v} / \partial \mathbf{x} - \partial \mathbf{u} / \partial \mathbf{y} \right) \qquad (9)$$

By assumption 1 above

$$\frac{\partial \zeta}{\partial \mathbf{x}} = \partial^2 \mathbf{v} / \partial \mathbf{x}^2 \qquad (10)$$

Put (10) into (8)

$$(\mathbf{u} - \mathbf{c}) \partial^2 \mathbf{v} / \partial \mathbf{x}^2 + \mathbf{v}\beta = 0 \qquad (11)$$

Differential Equation with solution of form

$$\mathbf{v} = \mathbf{A}\cos\frac{2\pi}{\mathbf{L}}(\mathbf{x} - \mathbf{ct}) \qquad (12)$$

Substitution of (12) into (11) and remembering definition of beta yields

$$c = u - \frac{\beta L^2}{4\pi^2} \qquad (13a)$$

$$\frac{\partial f}{\partial y} = \beta = \frac{\partial (2\Omega \sin\phi)}{\partial y} \qquad (i)$$

Rossby Wave Equation

A. <u>Assumptions</u>

1. Non-divergence
2. Zonal flow (or u>>v>>w and [∂(∂u/∂y)/∂x]=0

B. Rossby's Equation

Using barotropic vorticity equation, by assumption 1 above:

$$\frac{\mathbf{d}(\zeta + \mathbf{f})}{\mathbf{dt}} = 0 \qquad (1)$$

and

$$\frac{\mathbf{d}\zeta}{\mathbf{dt}} = -\frac{\mathbf{df}}{\mathbf{dt}} \qquad (2)$$

Expansion of the individual derivative on the right hand side of (2) and making appropriate deletions

$$\frac{df}{dt} = v\frac{\partial f}{\partial y} = v\beta \qquad (3)$$

Substitute (3) into (2) and expand left hand side.

$$\frac{\partial\zeta}{\partial \mathbf{t}} + \mathbf{u}\frac{\partial\zeta}{\partial \mathbf{x}} + \mathbf{v}\frac{\partial\zeta}{\partial \mathbf{y}} + \mathbf{w}\frac{\partial\zeta}{\partial \mathbf{z}} = -\mathbf{v}\beta \qquad (4)$$

By assumption 2 above

$$\mathbf{u}\frac{\partial\zeta}{\partial \mathbf{x}} >> \mathbf{v}\frac{\partial\zeta}{\partial \mathbf{y}} \qquad (5)$$

From last semester, in a situation of non-divergence, all of the local changes vorticity will be due to the translation of existing vorticity patterns because in a truly barotropic atmosphere there is no vorticity advection nor do air parcels experience a change in vorticity. This is expressed in equation (6).